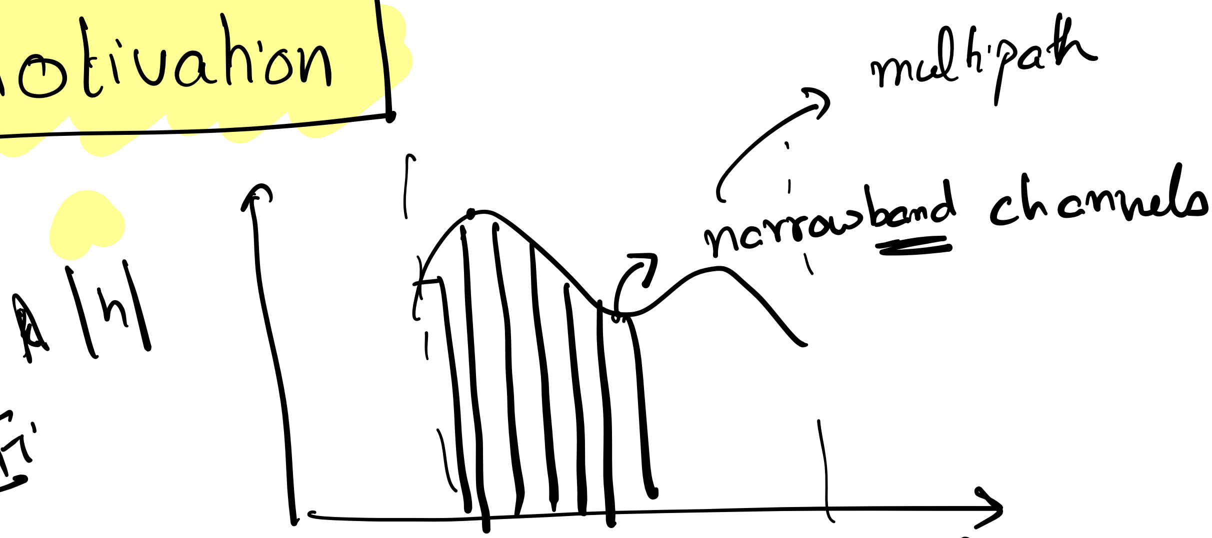
Today

- OFDM
  - Motivation
  - Packet Detection
  - Cyclic Prefix
  - CFO
  - Channel Estimation
  - Residual CFO & SFO

Motivation

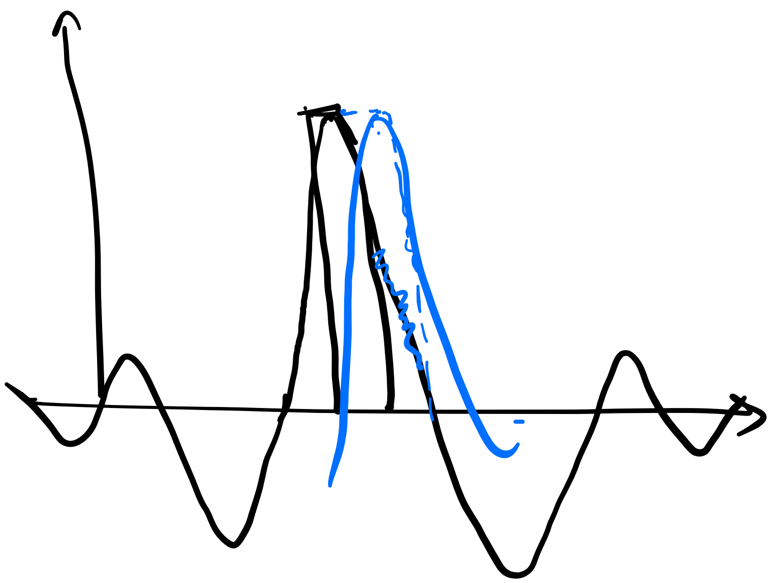


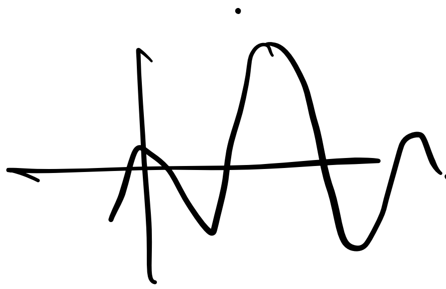
Wi-Fi

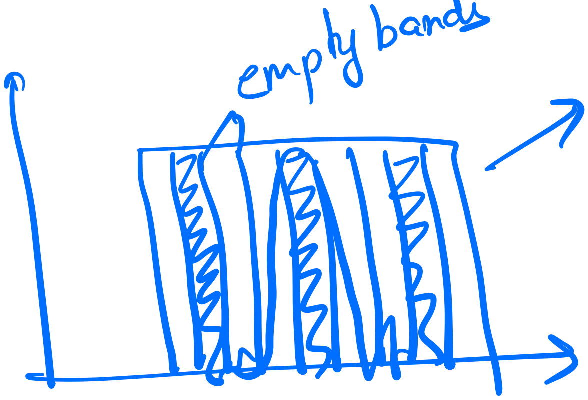
40MHz → 128 Subcarriers

20 MHz → 64 subcarrier

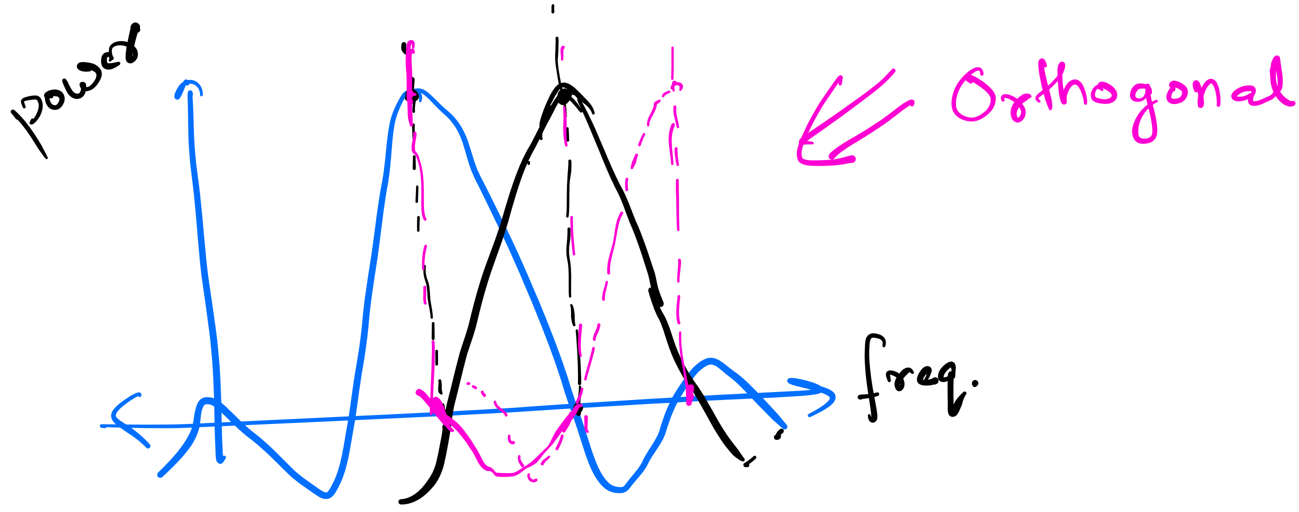
OFDM: Orthogonal Freq. Division Multiplexing

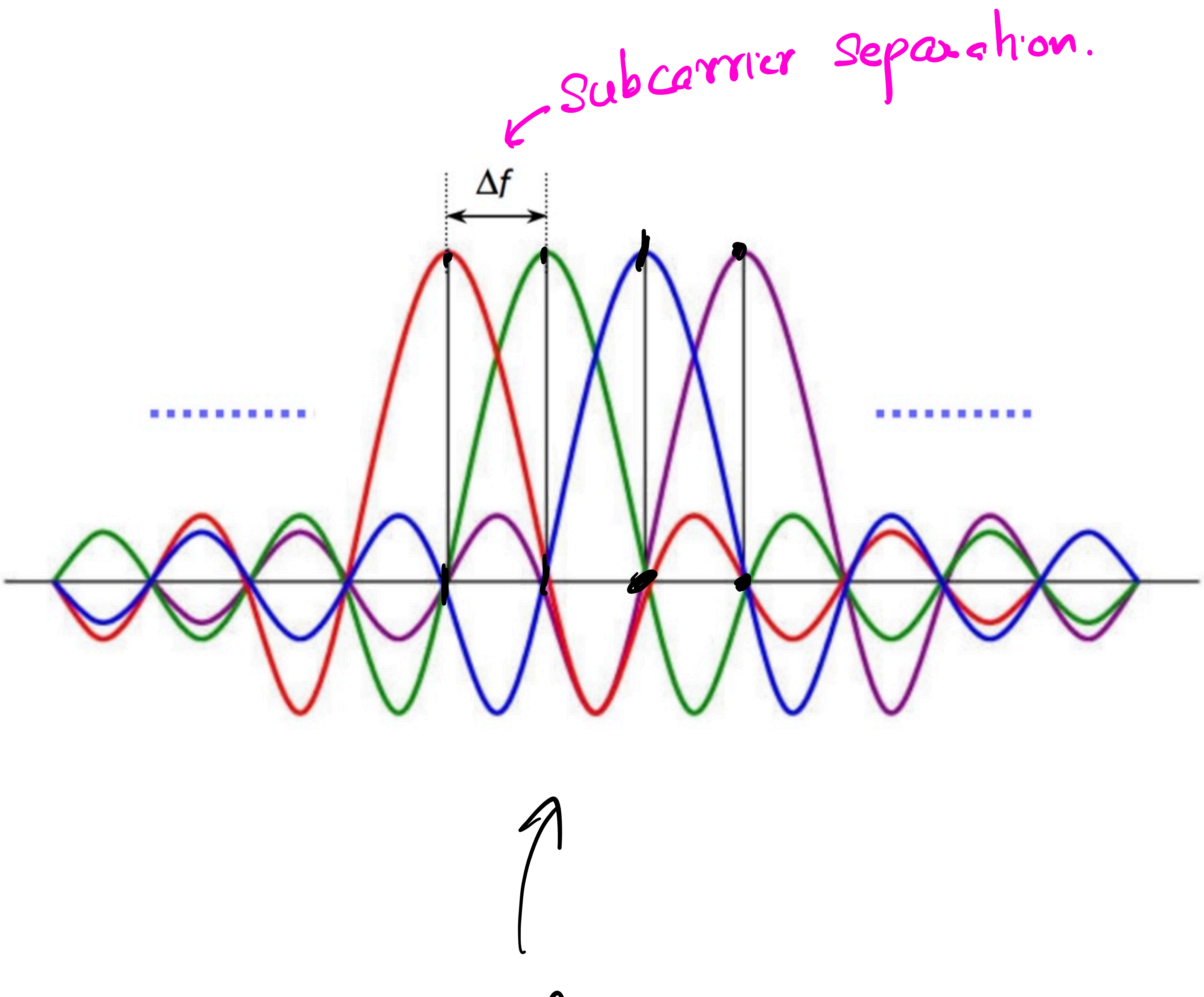






problem: lot of spectrum waste.


power
freq.
Orthogonal


Subcarrier separation.
Δf

Fourier Transform

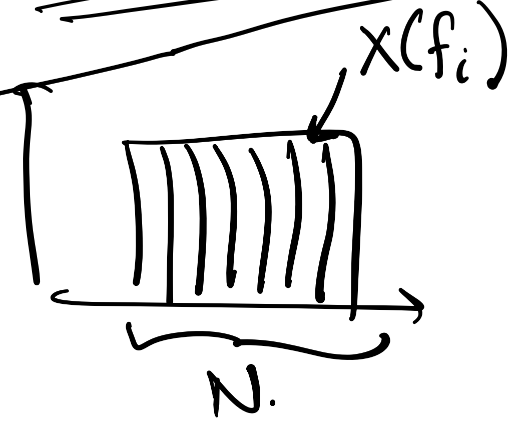
time domain ⇄ frequency domain

Discrete Fourier Transform

$x(t) \longrightarrow X(f)$ (N samples)

$$\rightarrow X(f_i) = \frac{1}{N}\sum_{t=0}^{N-1} x(t)\, e^{-j 2\pi f_i t / N} \quad \leftarrow \text{DFT}$$

$$x(t) = \sum_{i=0}^{N-1} X(f_i)\, e^{j 2\pi f_i t / N} \quad \leftarrow \text{IDFT}$$



1, 0, 1, ... 1, 1. ⟶ collect N samples ⟶ IDFT

↓

time domain signal

... 1, 1, 0, 1, 1 ⟵ DFT ⟵ collect N samp.

Pipeline

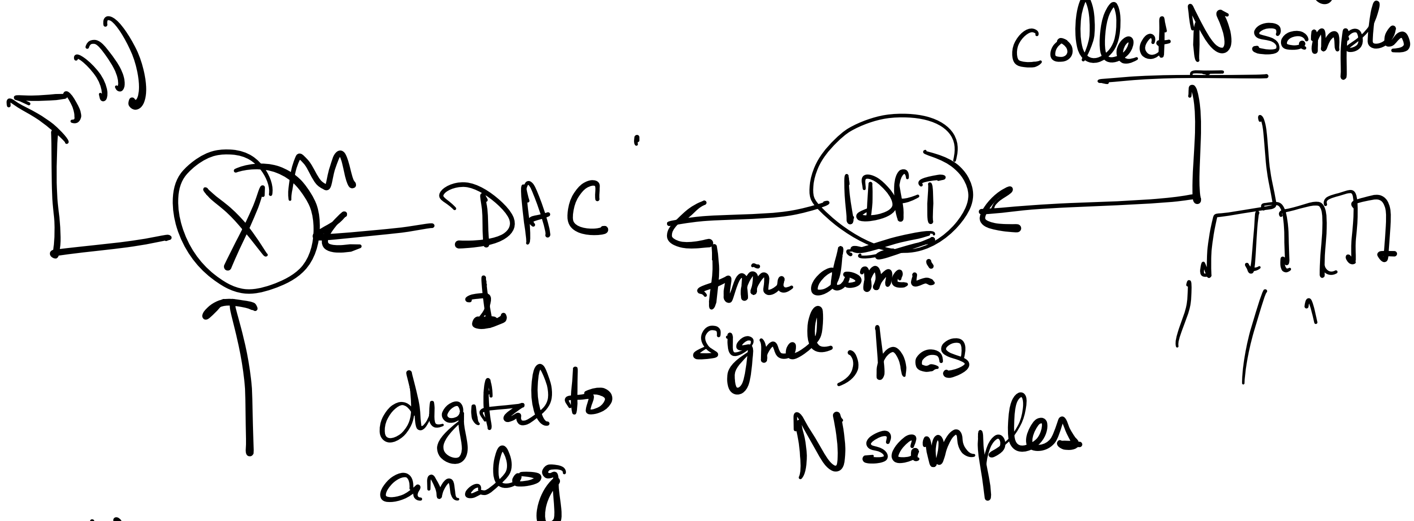
Transmitter.

BPSK
4-QAM
16-QAM

1, 0, 1, 1, . . . 11 —modulation→ -1, +1, -1, -1, . . . -1, -1

↓ freq. signal

collect N samples



Upconversion
[+fc] time dom
↳ add freq.

"complex math"

1Mbps
1024 → 1ms
1024

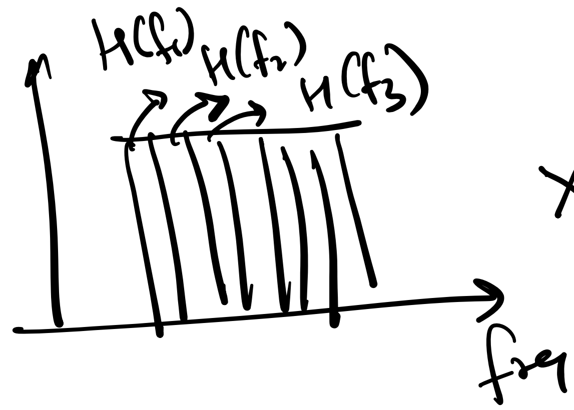
→ Bandwidth (20MHz), N-samples - IDFT

$$\frac{20MHz}{64} = \frac{20MHz}{N} \rightarrow 64 \leftarrow 64 \text{ samples}$$

Wi-Fi

Channel in freq. domain

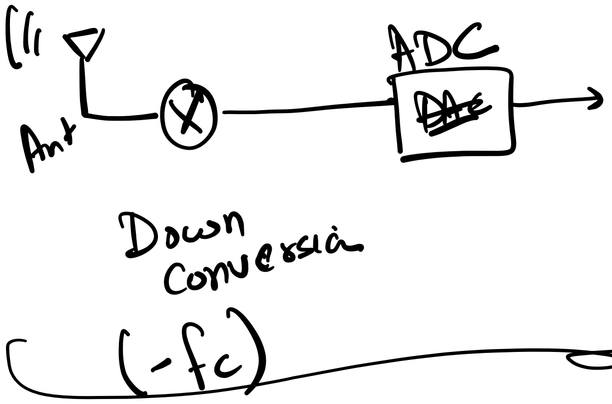
→



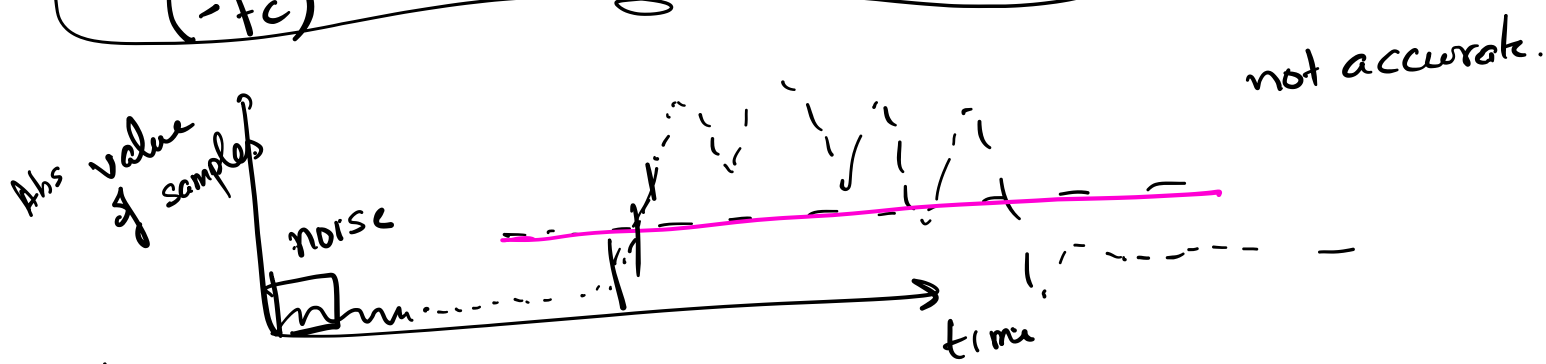
$X(f_1)$ $Y(f_1)$

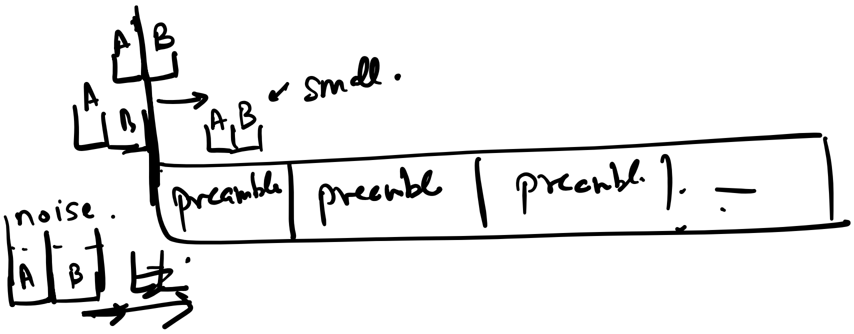
$$H(f_1) = \frac{Y(f_1)}{X(f_1)}$$

Packet Detection

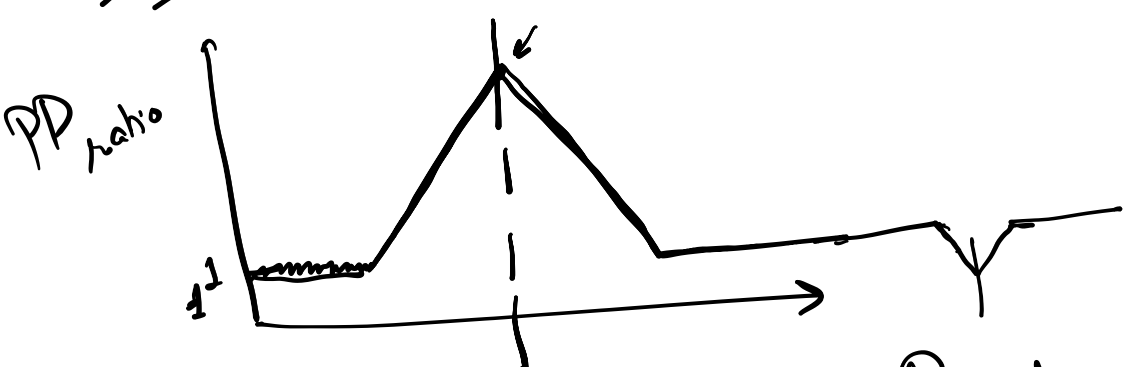


sequence of samples → bits?





$$PP_{ratio} = \frac{P_B}{P_A} = \frac{\sum_{i \in B} |x_i|^2}{\sum_{i \in A} |x_i|^2}$$

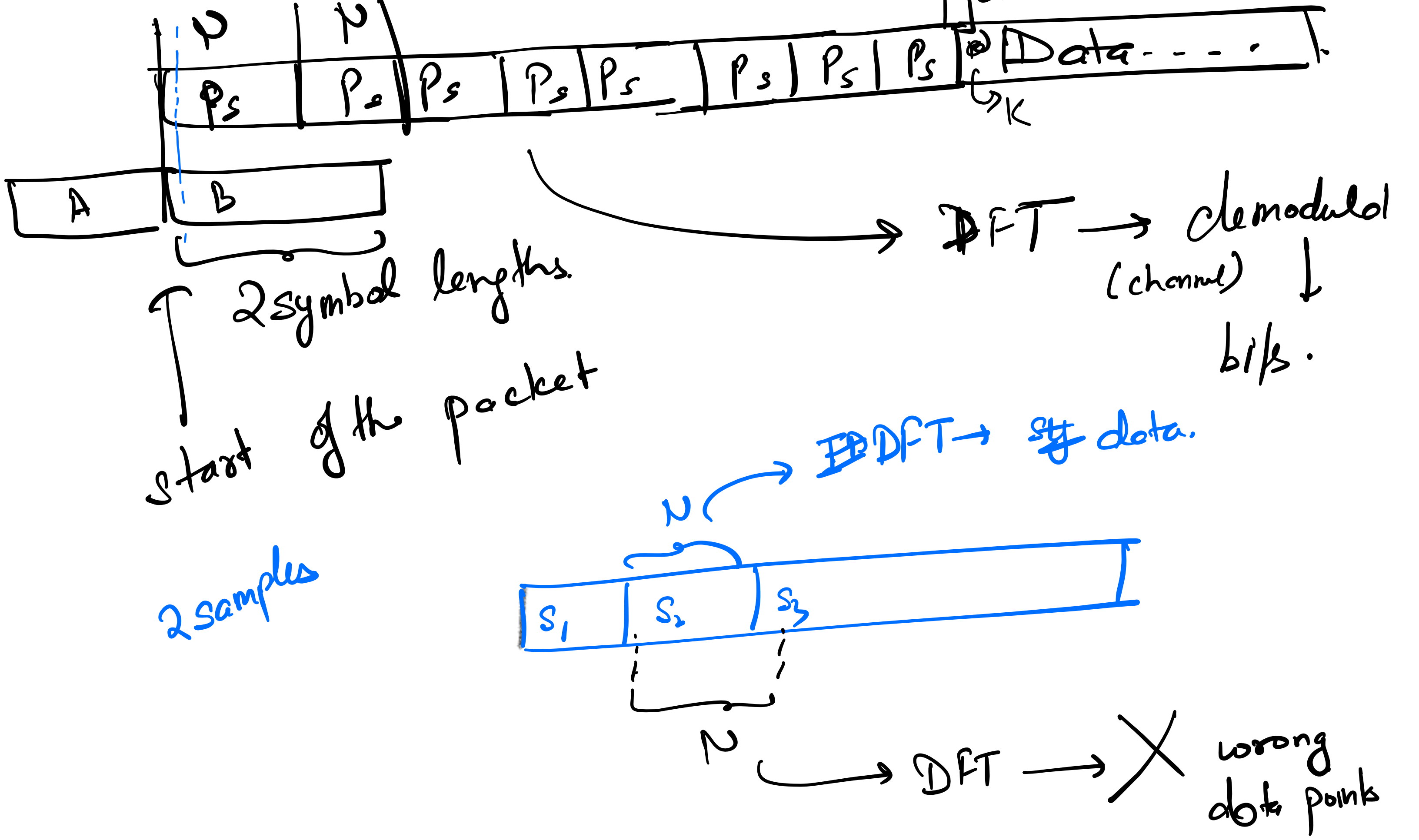


OFDM Packed detection

N narrowband freq → 1 OFDM symbol

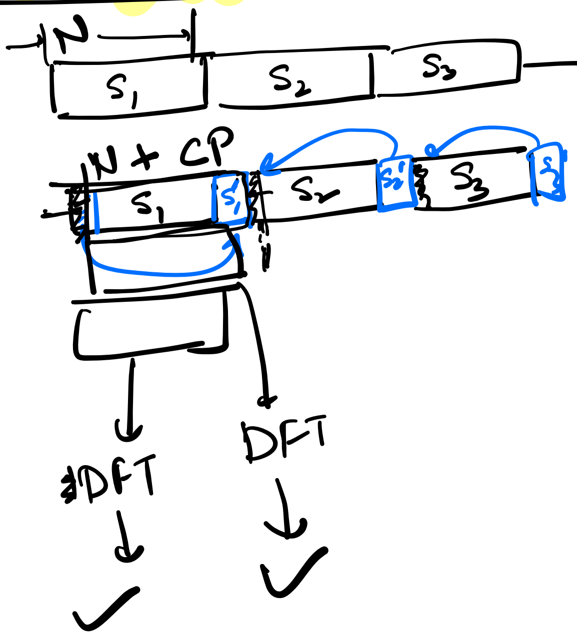
Wi-Fi, 64 samples → 1 OFDM symbol

DCP


N
N
Ps
Pe
Ps
Ps
Ps
Ps
Ps
Ps
Data - - - -
K
A
B
2 symbol lengths.
start of the packet
DFT → demodulat
(channel)
bits.
DFT → data.
N
2 samples
S1
S2
S3
N
DFT →
wrong data points

Cyclic Prefix

CP = length of cyclic prefix.
⇓
error tolerance of CP samples



$$x(t) \xrightarrow{DFT} X[f]$$

$$x(t-\delta) \rightarrow \boxed{X[f]} e^{-\frac{2\pi f \delta}{N}}$$

Wi-Fi → N = 64, 20MHz, CP = 16 ↳ 25%

$$\frac{64}{64+16} = \frac{64}{80} \times 100\% = 80\%$$

20%

LTE → N = 1024, CP = 72

$$\frac{72}{72+1024} \rightarrow \text{overhead} = 6.5\%$$

good CP
- → error tolerance
- → ISI → inter symbol interference

bad CP
- ↳ overhead

0.5 CP into the packet or 0.75 CP

Carrier Frequency Offset

- Intuition
- Correction

Upconversion at sender $[+f_c]$ — downconversion $[-f_c']$

$$x(t)\,e^{+j2\pi f_c t}$$

↓ clocks

$$y(t)\,e^{-j2\pi f_c' t}$$

↕ clocks

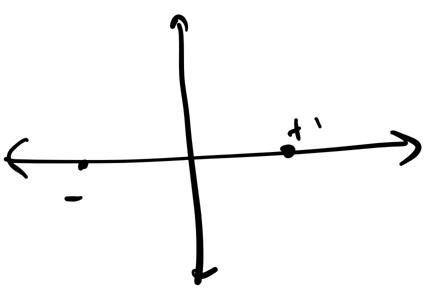
$$y(t) = x(t)\,e^{+j2\pi f_c t}\,e^{-j2\pi f_c' t}$$

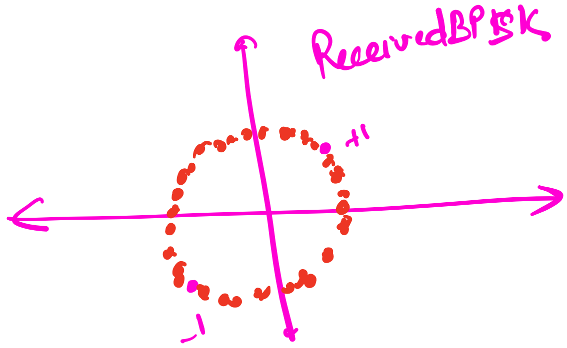
$$= x(t)\,e^{j2\pi (f_c - f_c') t}$$

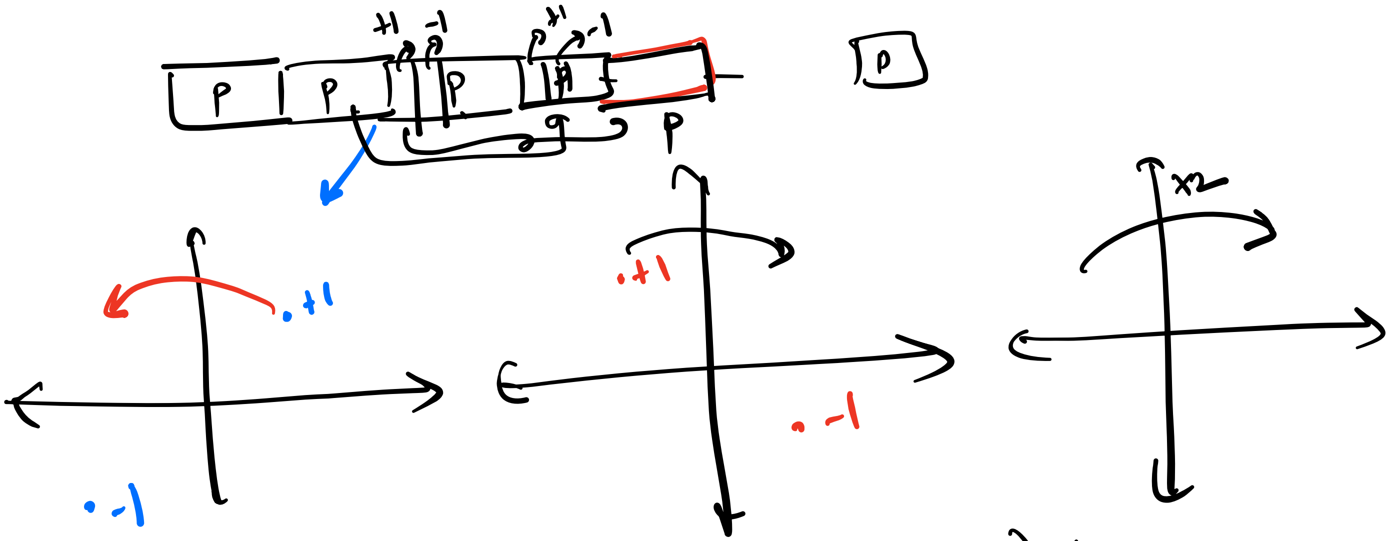
⇓

CFO or carrier freq. offset

BPSK







$$x(t)\, e^{j 2\pi \underline{\underline{(f_c - f_c')}}\, t}$$

$$\Delta f$$

$$y_1(t) = x(t)\, e^{+j 2\pi \Delta f\, t}$$

$$y_2(t) = x(t)\, e^{+j 2\pi \Delta f (t+N)}$$

$$= y_1(t+N)$$

$$A = \sum_{t=1}^{N} y_1(t)\, y_2^*(t) = \sum |x(t)|^2 e^{-j 2\pi \Delta f N}$$

$$\angle A = -j\, 2\pi\, \Delta f\, N$$

$$\boxed{y(t) * e^{-j \cdot 2\pi \Delta f N}}$$

A: $y_1(t) = x(t)\, e^{j2\pi\Delta f t}$

$y_2(t) = x(t)\, e^{j2\pi\Delta f(t+N)}$

$\dfrac{y_2(t)}{y_1(t)} = e^{j2\pi\Delta f N}$ $\neq 0$

$2\pi\Delta f N = \Phi$

$y_1(t)\, y_2^*(t) = \sout{x(t)\, x^*(t)}\, e^{j2\pi\Delta f t}$ (* → negative)

$|x(t)|^2\, e^{-2\pi\Delta f(t+N)}$

$|x(t)|^2\, e^{-j2\pi\Delta f N}$

Channel Estimation

Residual CFO/SFO

Summary

At TX:

- Create preamble symbol from training sequence (Uses BPSK)
- Repeat preamble symbol:
  - 4 times for packet detection
  - 2 times for CFO estimation
  - 2 times for channel estimation
  - Add CP for the last preamble
- Create data symbol from:
  - Data bits (Uses BPSK, QPSK, M-QAM)
  - Pilot bits (Uses BPSK)
- Add cyclic prefix to data symbols.

At RX:

- Detect beginning of packet.
- Estimate & correct for CFO.
- Jump $\approx 0.75\ CP$ samples into symbol to avoid ISI
- Estimate the channel.
- For each subsequent data symbol:
    - Remove CP
    - Take FFT of Size N
    - Correct for channel by dividing with $\widetilde{H}(f)$
    - Use linear regression to estimate residual CFO and SFO
    - Estimate accumulated phase $\Delta\phi(f)$ for each frequency bin
    - Add $\Delta\phi(f)$ to channel estimate $\widetilde{H}(f)$
    - Decode Bits